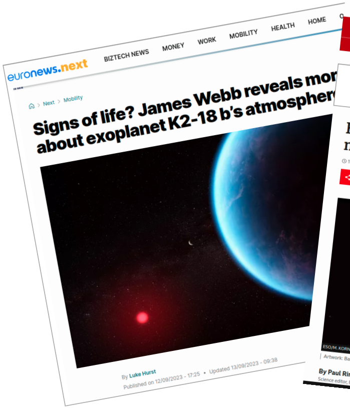
Direct Exoplanet Detection Using L1 Norm Low-Rank Approximation

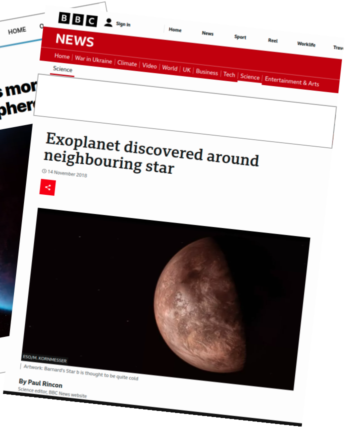
Hazan Daglayan[1], Simon Vary[1], Valentin Leplat[2], Nicolas Gillis[3], and P.-A. Absil[1]

[1] ICTEAM/UCLouvain, Belgium
[2] Center of Artificial Intelligence Technology, Skoltech, Russia
[3] University of Mons, Belgium

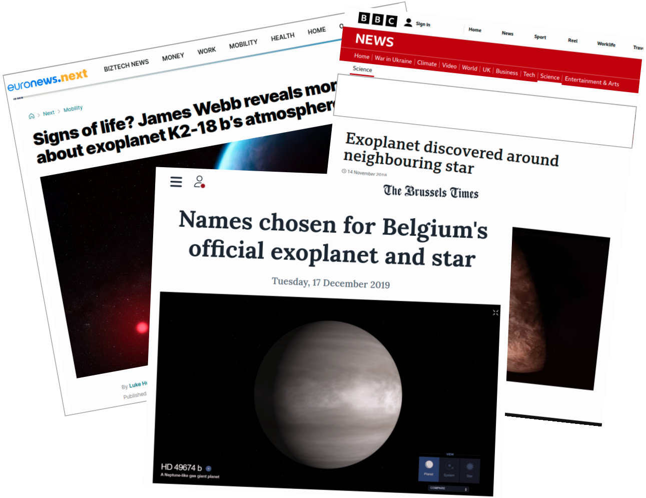
Exoplanet imaging

Exoplanet imaging

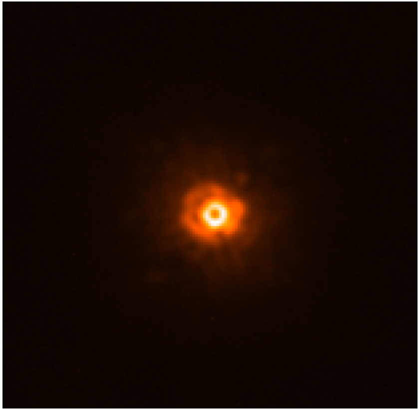




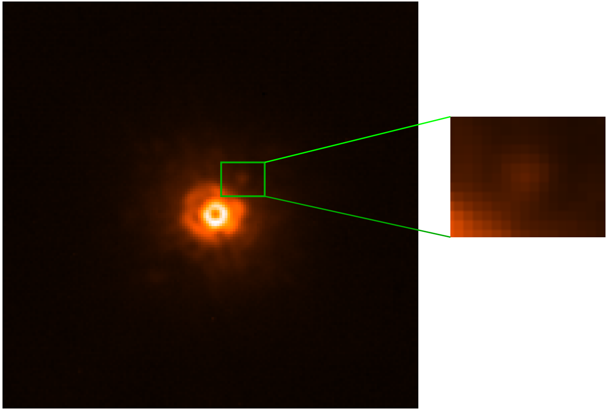
Exoplanet imaging



A real image



A real image of exoplanet



Direct imaging

Credit: https://exoplanets.nasa.gov/

Direct imaging

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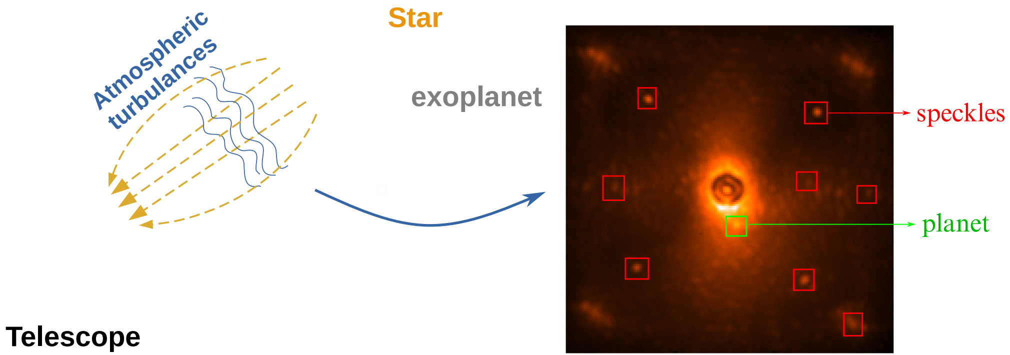
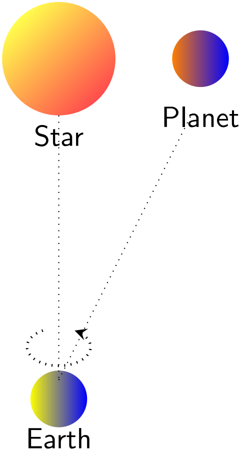
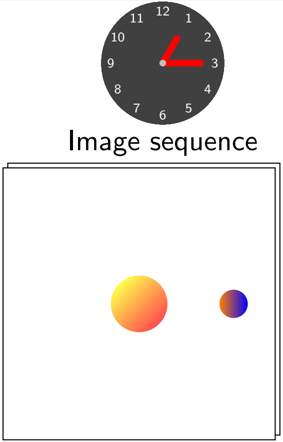
- firefly → exoplanet
- lighthouse → star

Direct Imaging

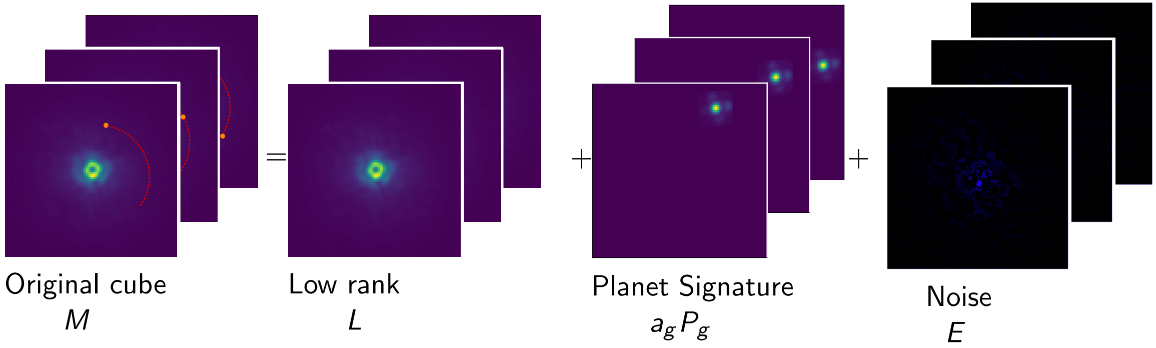


Angular differential imaging

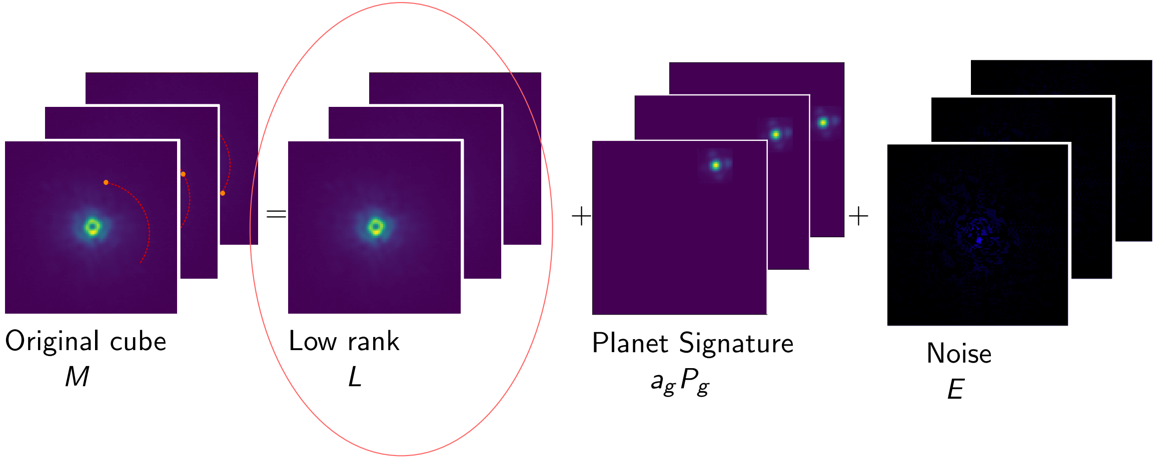




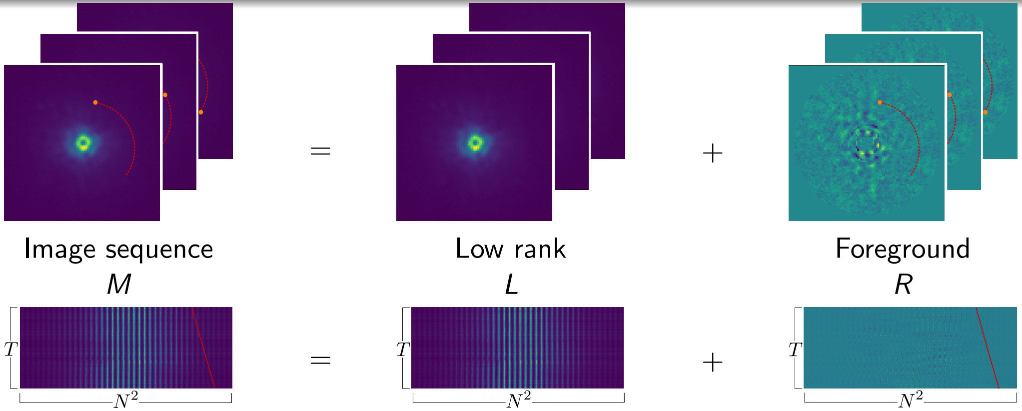
Problem setup & goal



Problem setup & goal



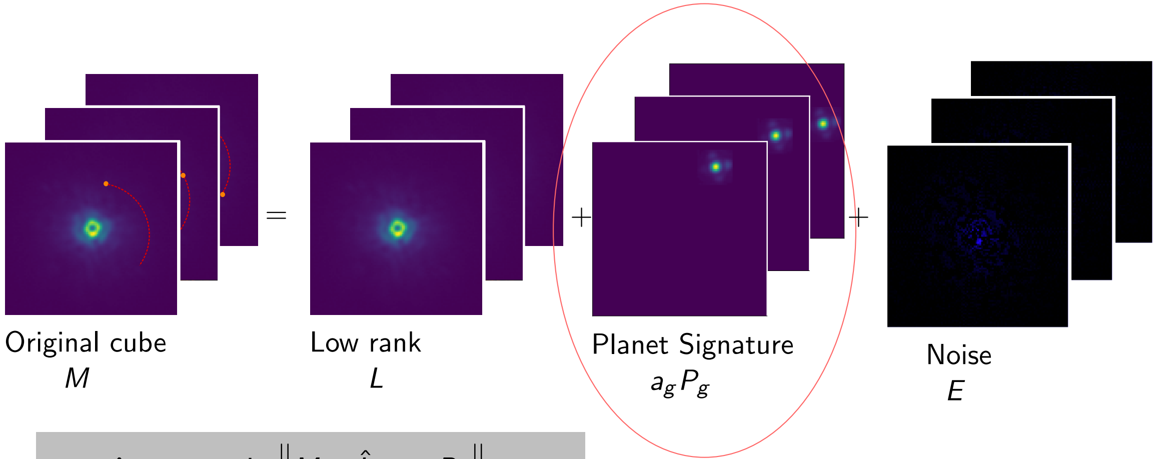
Background: (annular) PCA[1,2]



$$\hat{L} = \arg\min_{L} \|M - L\|_F \quad \text{subject to} \quad \mathrm{rank}(L) \leq k$$

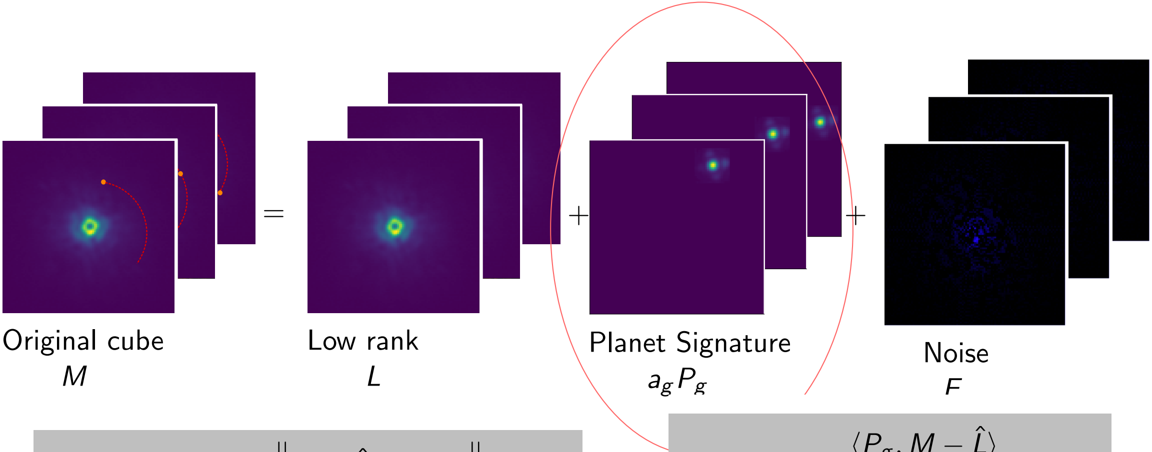
[1]Amara and Quanz, 2012
[2]Soummer, et al., 2012

The intensity of planet signature



$$\hat{a}_g = \arg\min_{a_g > 0} \left\| M - \hat{L} - a_g P_g \right\|_2 ,$$

The intensity of planet signature



$$\hat{a}_g = \arg\min_{a_g > 0} \left\| M - \hat{L} - a_g P_g \right\|_2 ,$$

$$\hat{a}_g = \frac{\langle P_g , M - \hat{L} \rangle}{\| P_g \|_F^2}$$

The intensity of planet signature

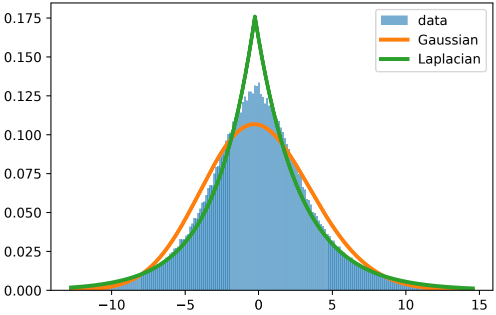


Figure: Residual data at $10\lambda/D$ [3,4]

[3] Pairet, et al., 2019

[4] Daglayan, et al., 2022

The intensity of planet signature

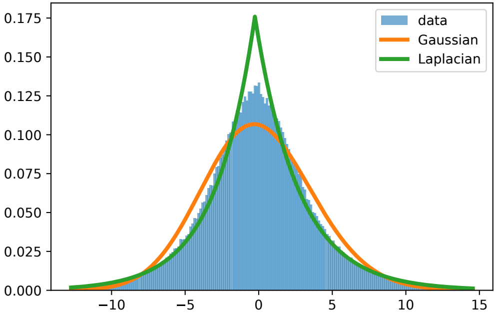


Figure: Residual data at $10\lambda/D$ [3,4]

$$\hat{a}_g = \arg\min_{a_g > 0} \left\| M - \hat{L} - a_g P_g \right\|_1$$

[3] Pairet, et al., 2019

[4] Daglayan, et al., 2022

The intensity of planet signature

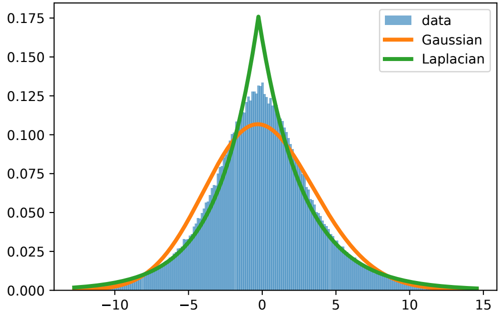
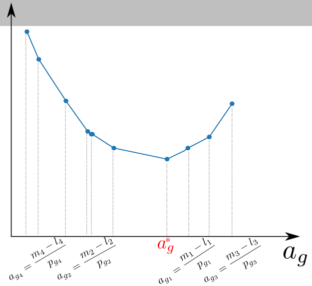


Figure: Residual data at $10\lambda/D$ [3,4]

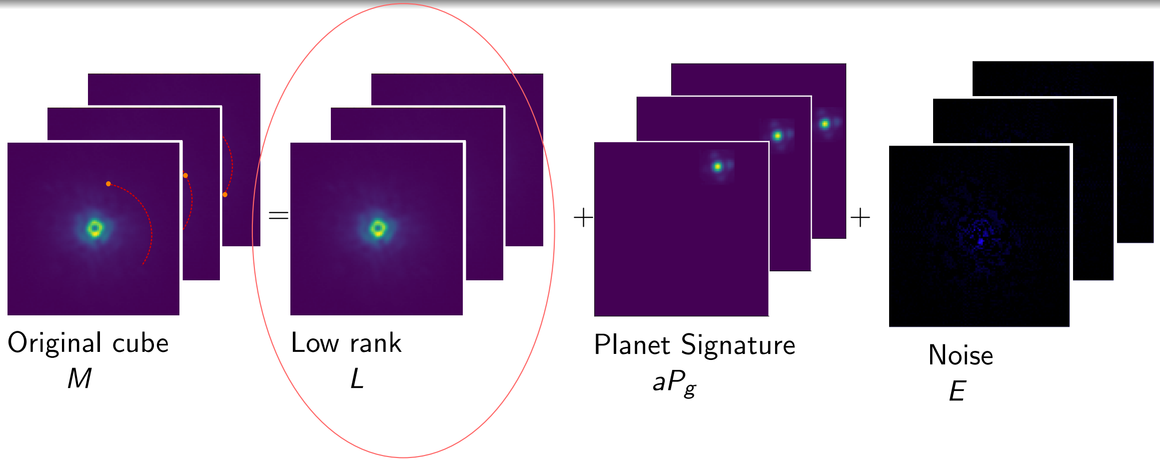
$$\hat{a}_g = \arg\min_{a_g>0} \left\| M - \hat{L} - a_g P_g \right\|_1$$

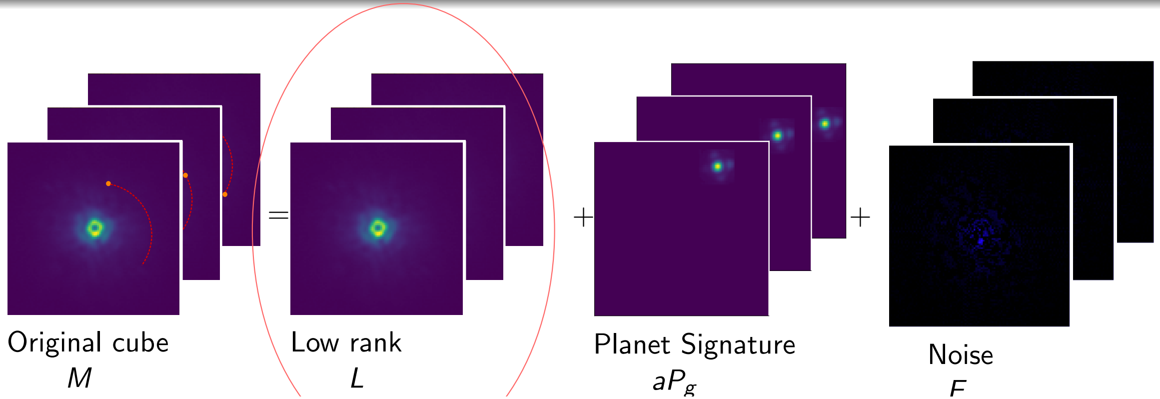
$$\hat{a}_g = \arg\min_{a_g\in\mathbb{R}} \sum_j \left| m_j - l_j - a_g p_{g_j} \right|$$



[3] Pairet, et al., 2019
[4] Daglayan, et al., 2022

Background: (annular) L1-LRA





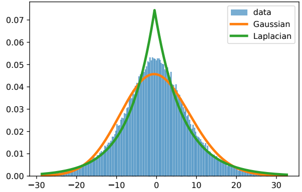
$$\hat{L} = \arg\min_{L} \|M - L\|_1 \quad \text{subject to} \quad \mathrm{rank}(L) \leq k$$

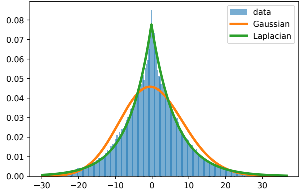
- Solved using an exact block-cyclic coordinate descent method[5].

[e] Gillis and Plemmons, 2011

Data distributions of residual matrix after PCA/L1-LRA

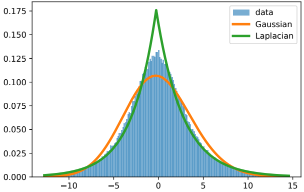
PCA
at $4\lambda/D$

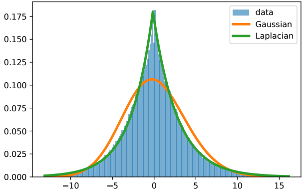




L1-LRA
at $4\lambda/D$

PCA
at $10\lambda/D$





L1-LRA
at $10\lambda/D$

Coefficient of determination

$$\rho^2 = \frac{\left(\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})\right)^2}{\left(\sum_{i=1}^{n}(x_i - \bar{x})^2\right)\left(\sum_{i=1}^{n}(y_i - \bar{y})^2\right)}$$

- x_i : the height of the bins of the data histogram
- y_i : the values of the probability density function within these bins
- $\bar{x}$ and $\bar{y}$: mean of x_i and y_i, respectively

Table: The coefficient of determination ρ^2 for large separation $10\lambda/D$

Rank	**PCA**		**L1-LRA**	
	Gaussian	**Laplacian**	**Gaussian**	**Laplacian**
5	0.9866	0.9920	0.9894	**0.9940**
10	0.9859	0.9936	0.9872	**0.9948**
15	0.9912	0.9954	0.9922	**0.9960**

L1 Norm[6]

$$\log \Lambda_g(R) = -\sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)| - |R(t,r)|}{\sigma_{R(r)}} \tag{1}$$

L2 Norm

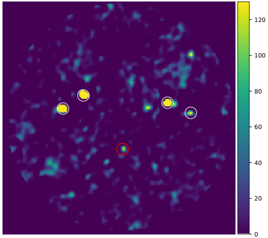
$$\log \Lambda_g(R) = -\frac{1}{2} \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)|^2 - |R(t,r)|^2}{\sigma^2_{R(r)}} \tag{2}$$

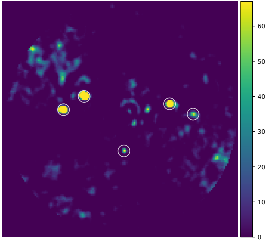
Background subtraction	**Planet detection**	
	L1 (2)	L2 (1)
L1-LRA	L1L1	L1L2
PCA	L2L1	L2L2

[6]Daglayan, et al., 2022

Likelihood ratio map results

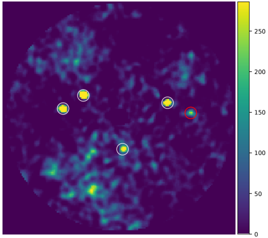
L2L2

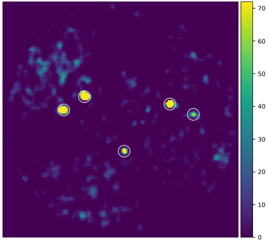




L2L1

L1L2

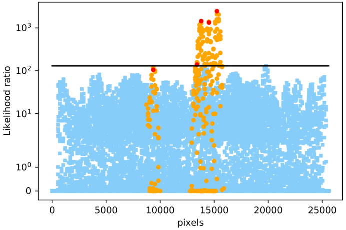


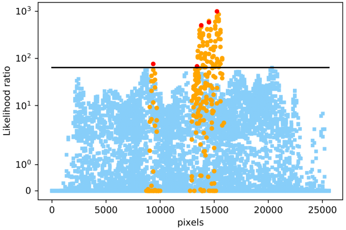


L1L1

Likelihood ratio map results

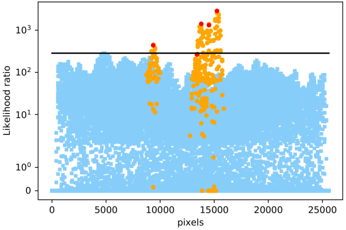
L2L2

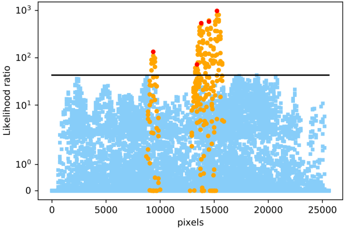




L2L1

L1L2





L1L1

ROC curve results

- $\sqrt{\text{TPR}}$ & $\sqrt{\text{FPR}}$ are used instead of TPR & FPR.
- 50 different datasets are used by injecting two synthetic planets in each, 180 degrees apart.
- Planets are placed at the separation $10\lambda/D$.

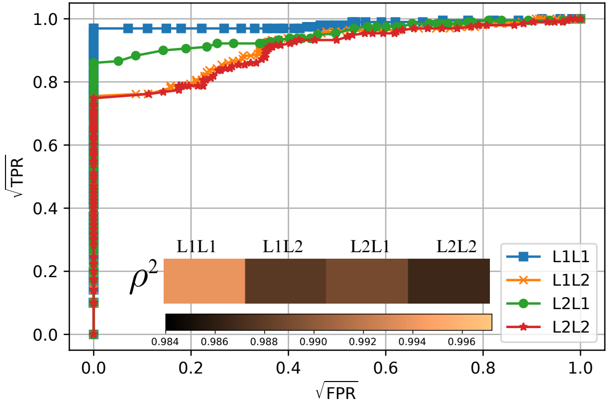


Figure: ROC Curves - Likelihood ratio map

ROC curve results

- $\sqrt{\text{TPR}}$ & $\sqrt{\text{FPR}}$ are used instead of TPR & FPR.
- 50 different datasets are used by injecting two synthetic planets in each, 180 degrees apart.
- Planets are placed at the separation $4\lambda/D$.

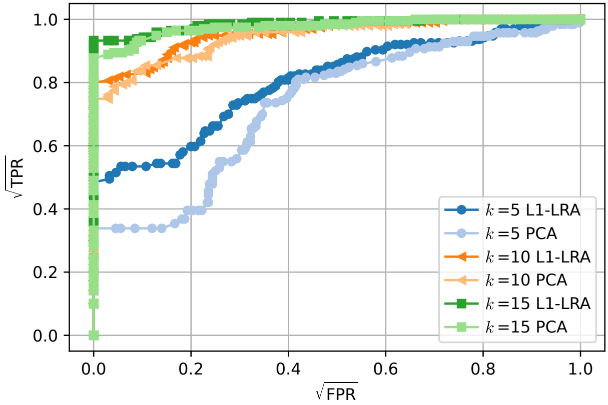
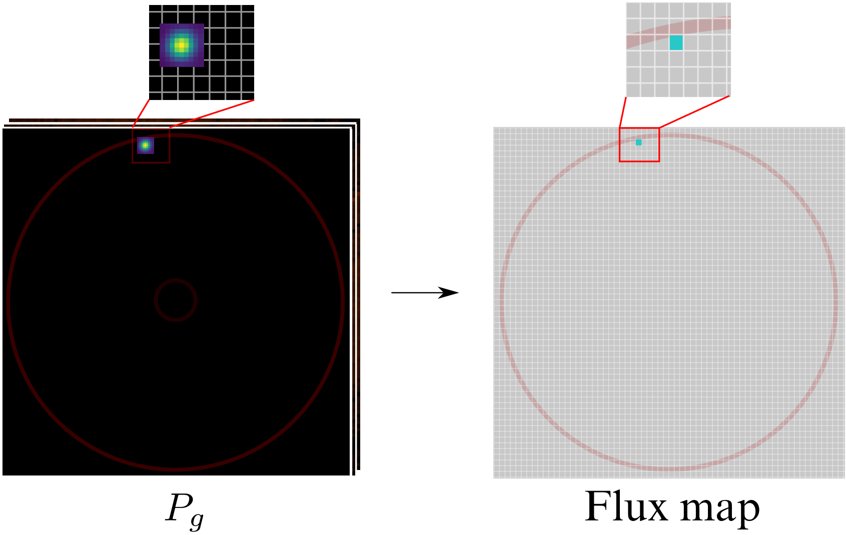


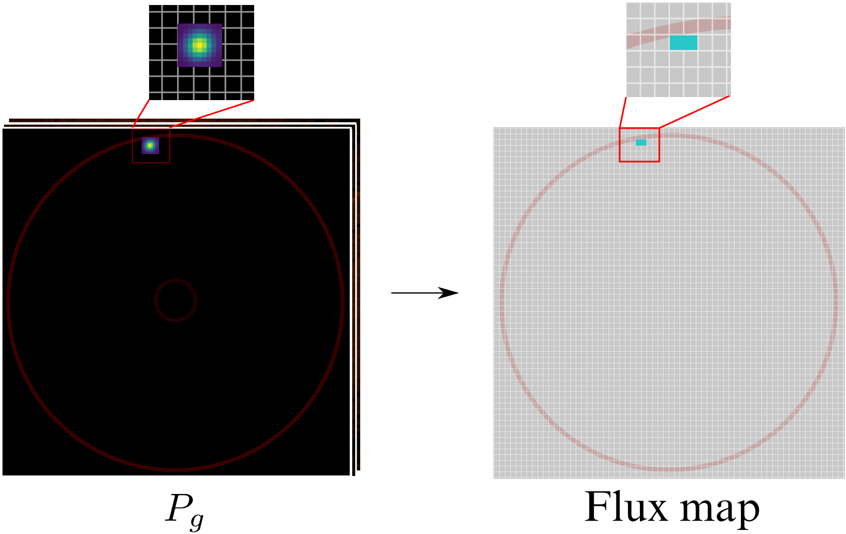
Figure: ROC Curves - SNR map

Thank you for your attention!
Any questions?

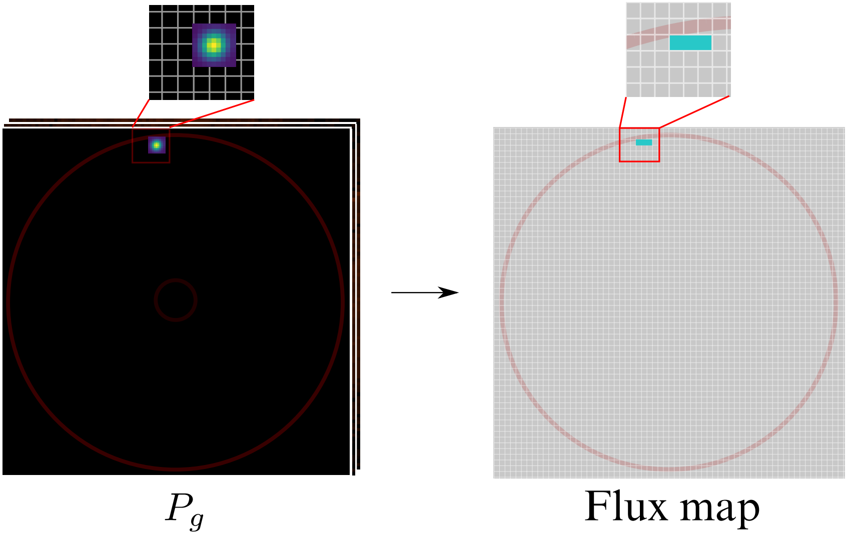
hazan.daglayan@uclouvain.be
GitHub: hazandaglayan/l1lra_for_exoplanets

Trajectories

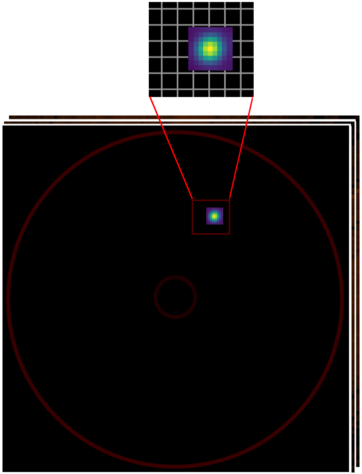



P_g
Flux map

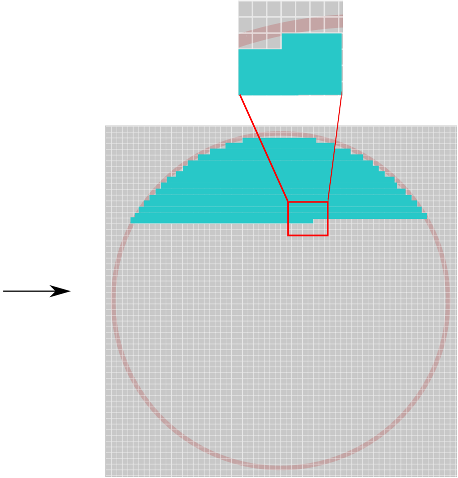
Trajectories



Trajectories



P_g



Flux map

More ROC curves