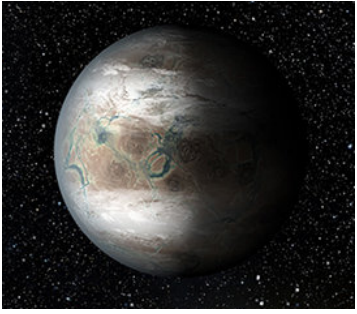
UCLouvain

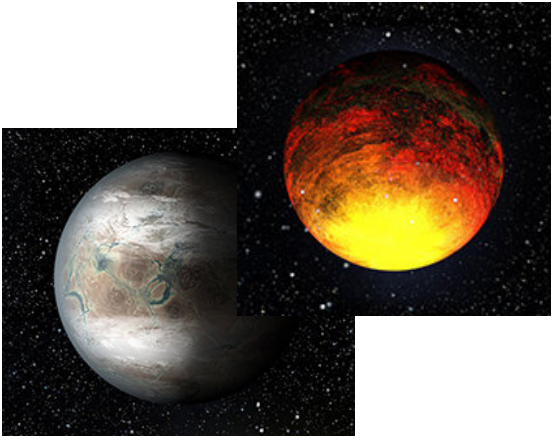
Likelihood Ratio Map for Direct Exoplanet Detection

Hazan Daglayan, Simon Vary, Faustine Cantalloube, P.-A. Absil, and Olivier Absil

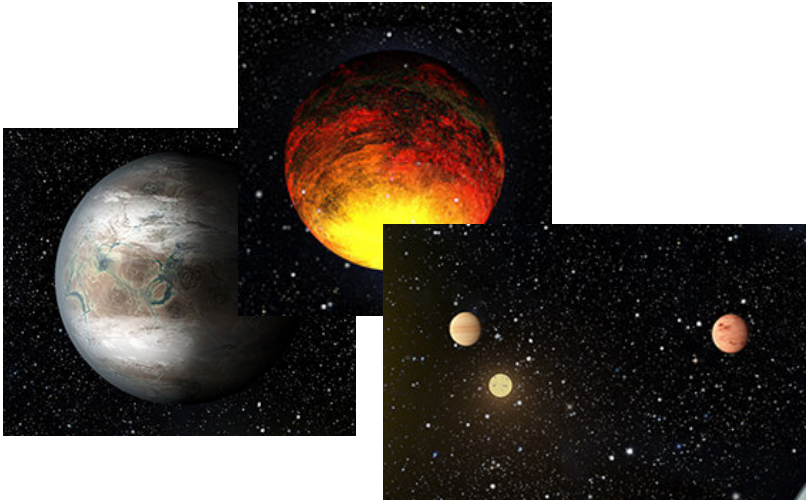
Exoplanet Imaging



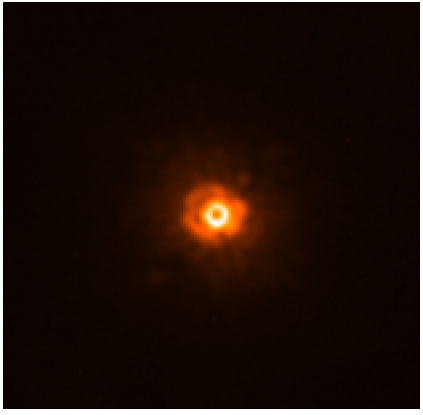
Exoplanet Imaging



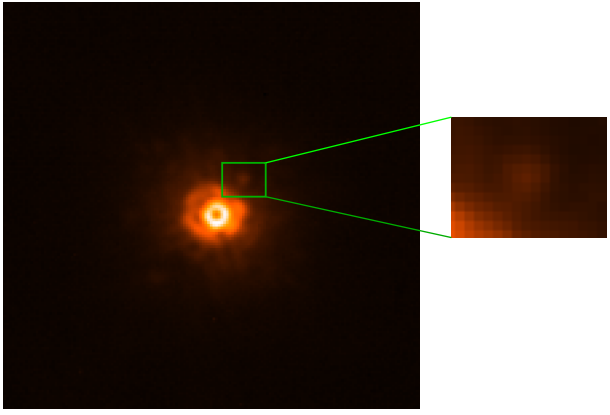
Exoplanet Imaging



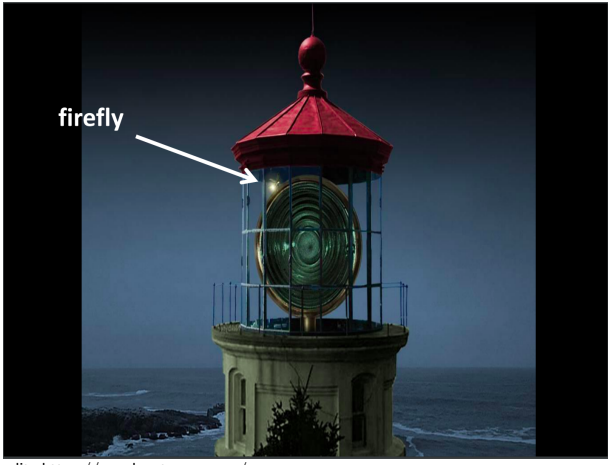
Exoplanet Imaging



Exoplanet Imaging



Direct Imaging



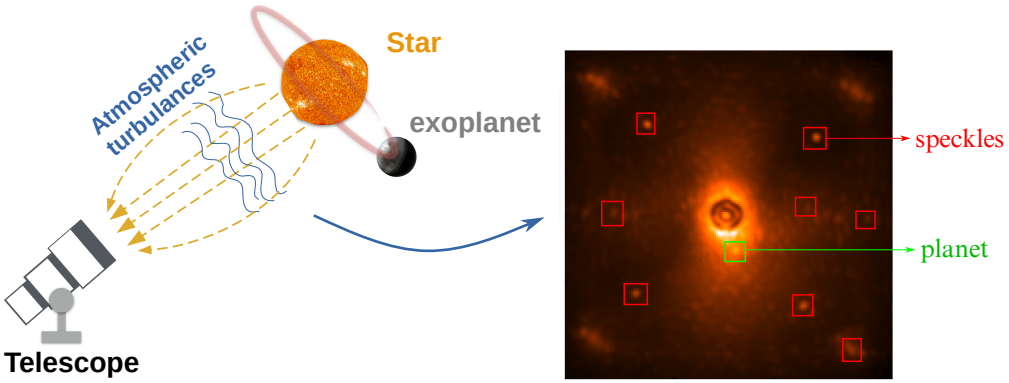
Credit: https://exoplanets.nasa.gov/

Direct Imaging

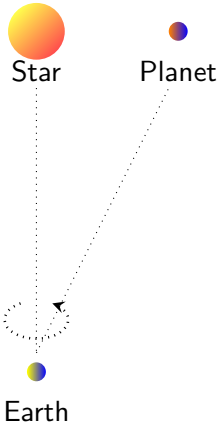
Credit: https://exoplanets.nasa.gov/

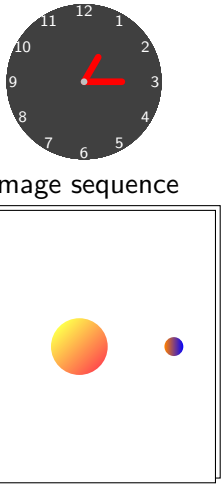
- firefly $\rightarrow$ exoplanet
- lighthouse $\rightarrow$ star

Direct Imaging



Angular Differential Imaging





Problem setup & goal

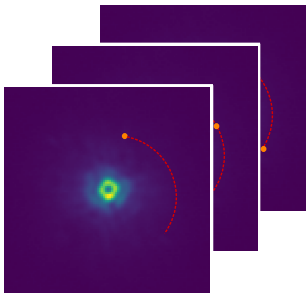
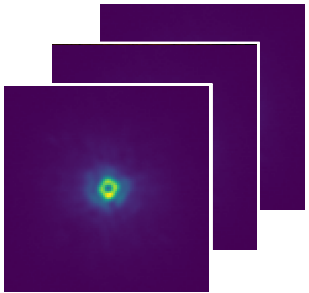


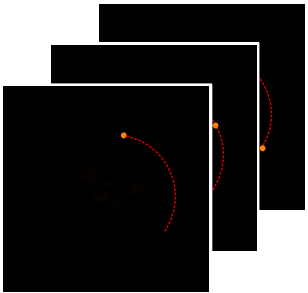
Image sequence

=



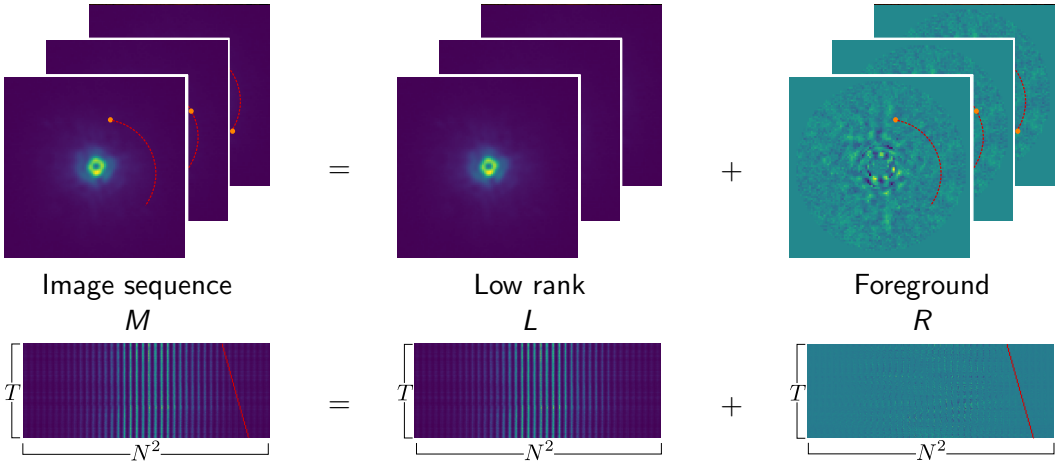
Background
(star+speckles)

+



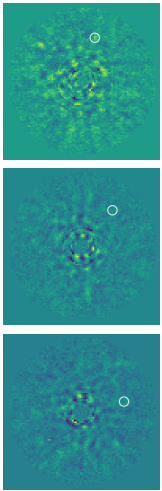
Foreground
(planet)

Background: (Annular) PCA[1,2]



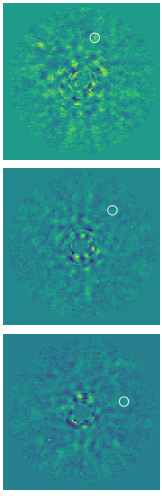
[1] Amara and Quanz, 2012
[2] Soummer, et al., 2012

State of art: median SNR map

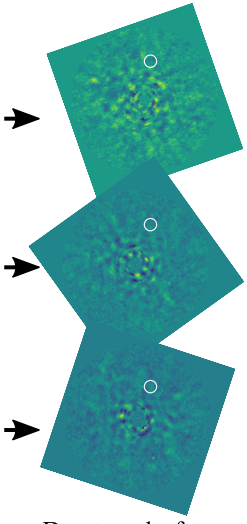


Foreground

State of art: median SNR map

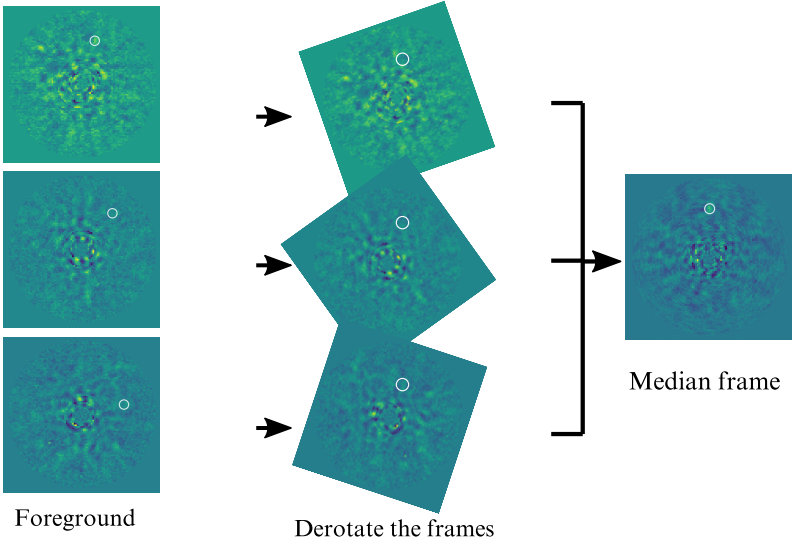


Foreground

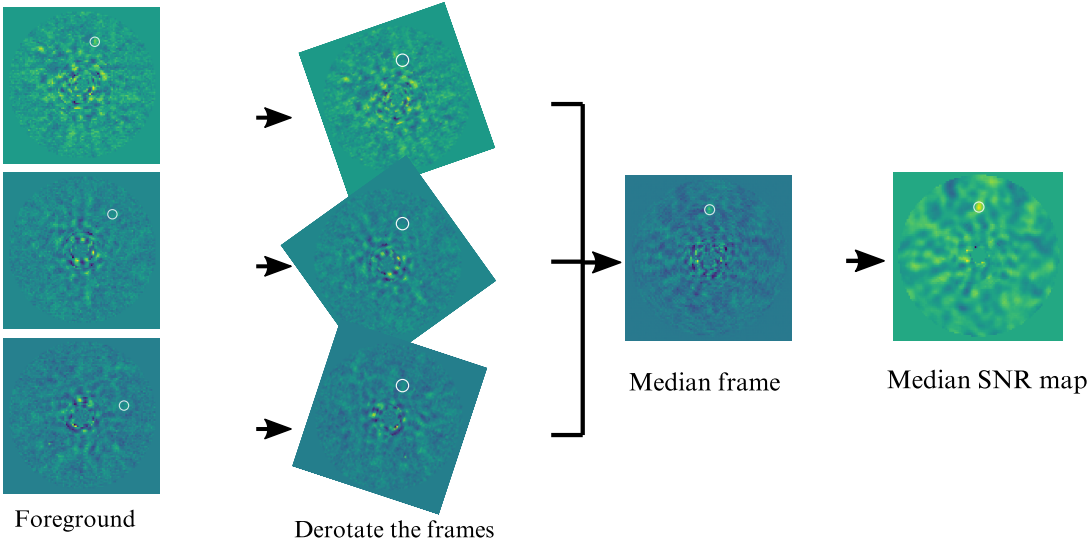


Derotate the frames

State of art: median SNR map

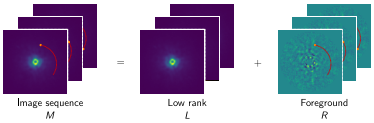


State of art: median SNR map

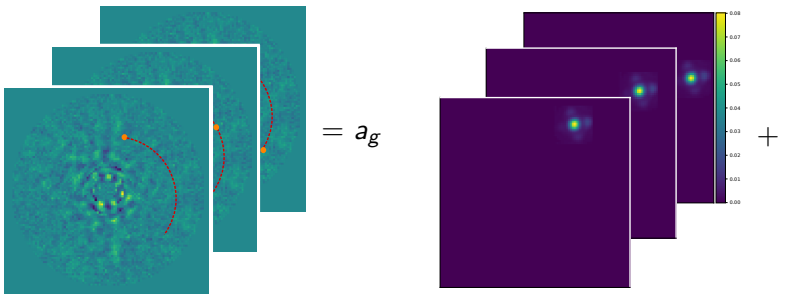


Statistical Model for Foreground

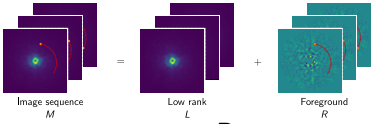
Model based on point spread function (PSF)



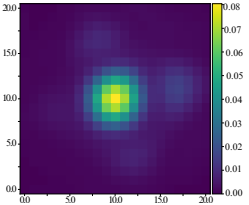
$$R = a_g P_g + E$$



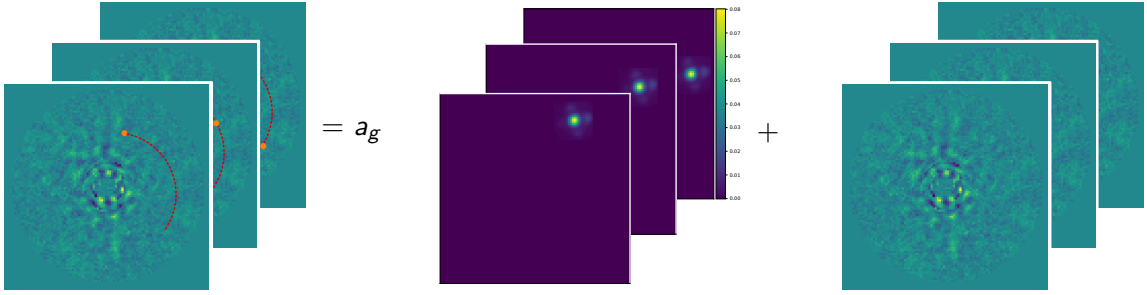
Model based on point spread function (PSF)



$$R = a_g P_g + E$$



Planet signature



Detection based on likelihood ratio map

Estimate the value of a_g by maximizing the log-likelihood

Log Likelihood under Gaussian Noise

$$\log \mathcal{L}_g^{\text{Gauss}}(a|R) \propto -\frac{1}{2} \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|^2}{\sigma^2_{R(r)}}, \tag{1}$$

[3] Pairet, 2019

Detection based on likelihood ratio map

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We replace the Gaussian assumption in (1) with a Laplacian [3]

Log Likelihood under Laplacian Noise

$$\log \mathcal{L}_g(a|R) \propto -\sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|}{\sigma_R(r)},$$

[3] Pairet, 2019

Detection based on likelihood ratio map

Estimate the value of a_g by maximizing the log-likelihood

Log Likelihood under Gaussian Noise

$$\log \mathcal{L}_g^{\text{Gauss}}(a|R) \propto -\frac{1}{2} \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|^2}{\sigma^2_{R(r)}}, \tag{1}$$

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MLE of planet's brightness

$$\begin{aligned}\hat{a}_g &= \arg\max_a \log \mathcal{L}_g(a|R) \\ &= \arg\min_a \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|}{\sigma_R(r)}.\end{aligned}$$

[3] Pairet, 2019

Detection based on likelihood ratio map

Likelihood ratio

$$\begin{aligned}\log \Lambda_g(R) &= \log\left(\frac{\mathcal{L}_g(\hat{a}_g|R)}{\mathcal{L}_g(0|R)}\right) \\ &= -\sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)| - |R(t,r)|}{\sigma_R(r)}.\end{aligned}$$

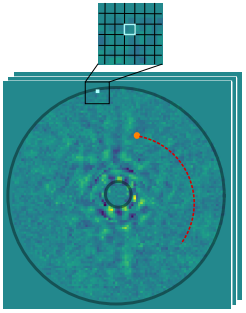
Trajectories

MLE of planet's brightness

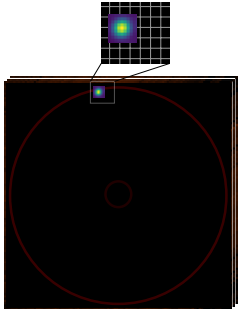
$$\hat{a}_g = \arg\min_a \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|}{\sigma_R(r)}$$

Likelihood ratio

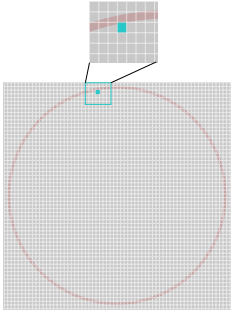
$$\log \Lambda_g(R) = -\sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)| - |R(t,r)|}{\sigma_R(r)}$$



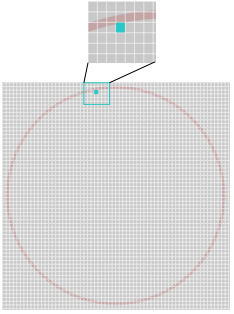
R



P_g



Flux frame



LR map

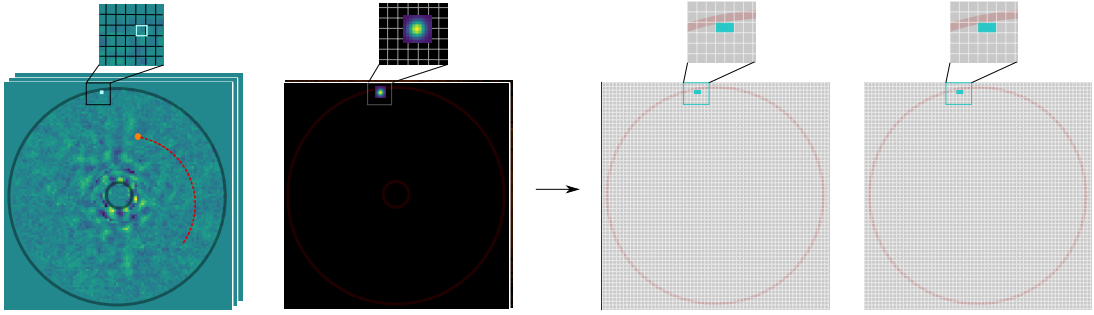
Trajectories

MLE of planet's brightness

$$\hat{a}_g = \arg\min_a \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|}{\sigma_R(r)}$$

Likelihood ratio

$$\log \Lambda_g(R) = -\sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - \hat{a}_g P_g(t,r)| - |R(t,r)|}{\sigma_R(r)}$$



R P_g Flux frame LR map

Trajectories

MLE of planet's brightness

$$\hat{a}_g = \arg\min_a \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|}{\sigma_R(r)}$$

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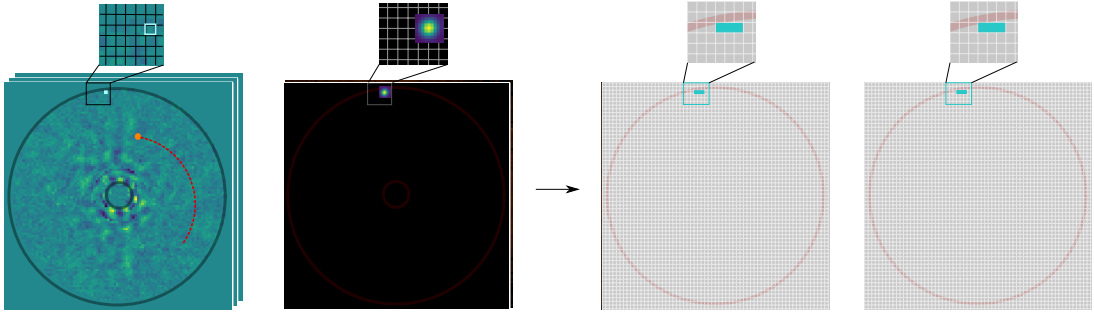

R P_g Flux frame LR map

Trajectories

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$$\hat{a}_g = \arg\min_a \sum_{(t,r)\in\Omega_g} \frac{|R(t,r) - aP_g(t,r)|}{\sigma_R(r)}$$

Likelihood ratio

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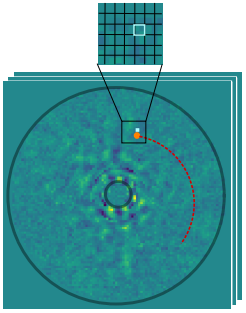
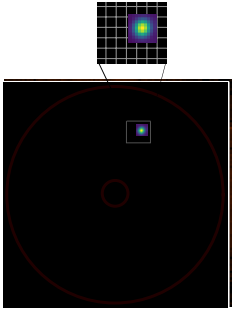
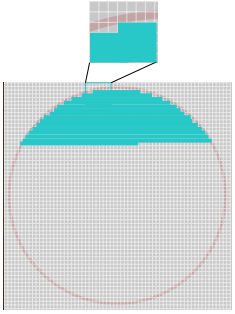


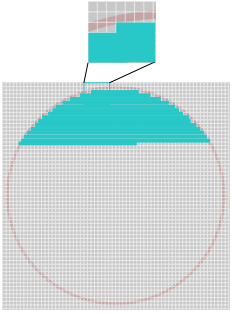
R



P_g



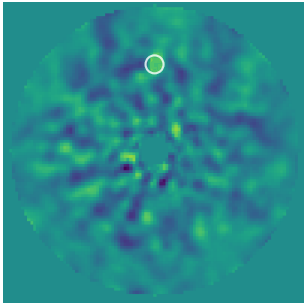
Flux frame



LR map

Likelihood ratio (LR) map

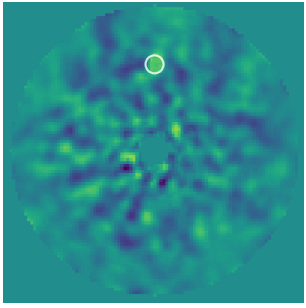
All fluxes a_g form flux frame.



Flux frame

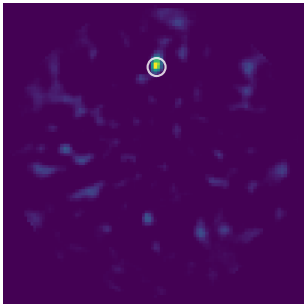
Likelihood ratio (LR) map

All fluxes a_g form flux frame.



Flux frame

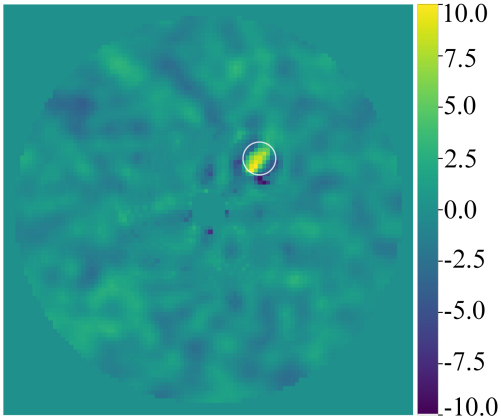
All log likelihood ratios $\log \Lambda_g(R)$ form likelihood map.



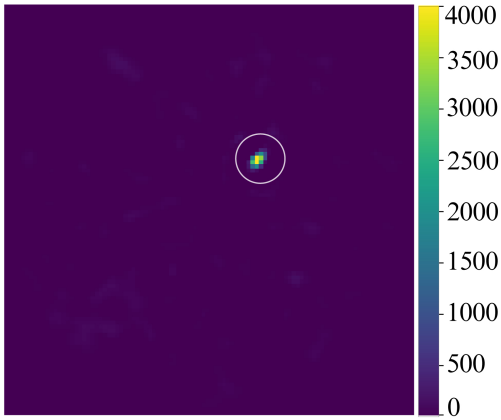
LR map

Detection Maps

Real planet:



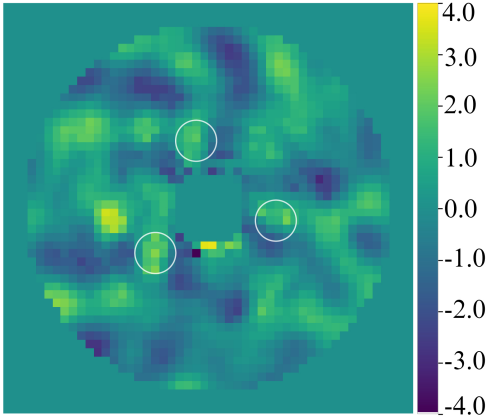
Median SNRmap



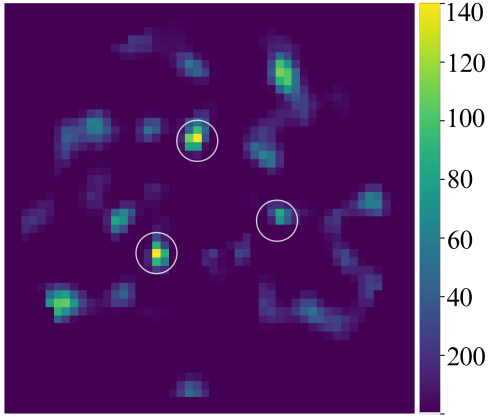
LRmap

Detection Maps

Synthetic planets:



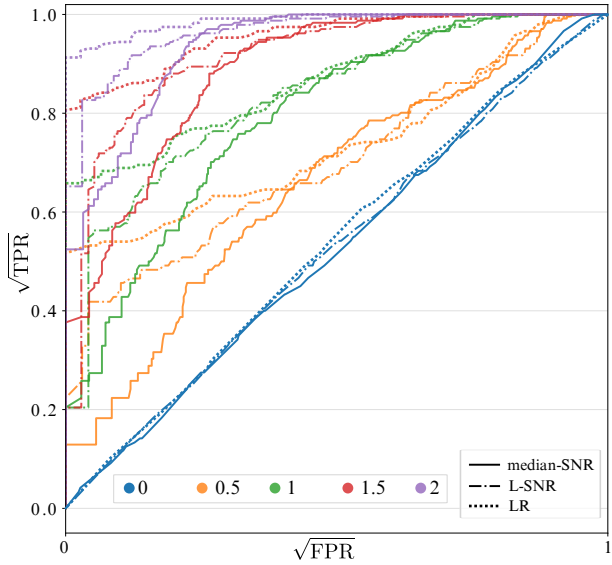
Median SNRmap



LRmap

ROC Curve Comparison

- Synthetic planets are injected.
- $\sqrt{\mathrm{TPR}}$ & $\sqrt{\mathrm{FPR}}$ are used instead of TPR & FPR.



Thank you for your attention!
Any questions?