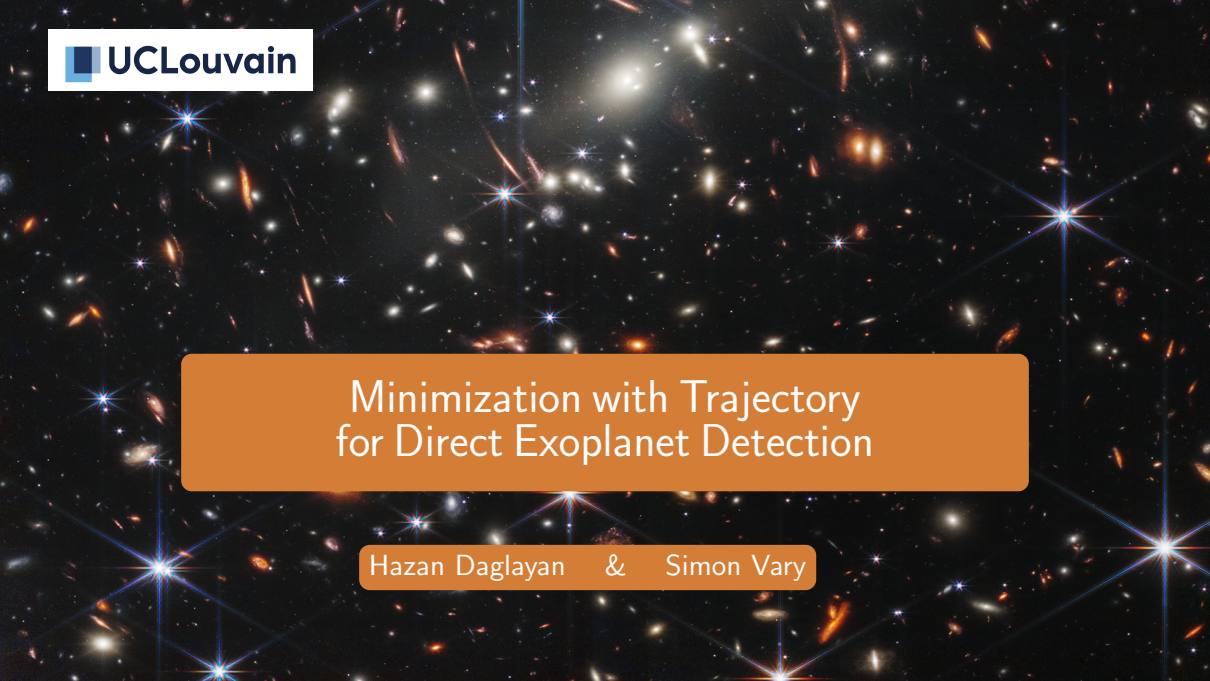
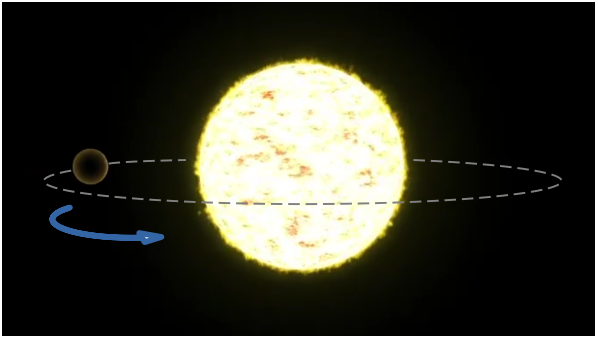
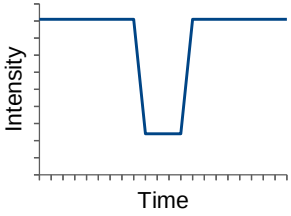

UCLouvain
Minimization with Trajectory
for Direct Exoplanet Detection
Hazan Daglayan & Simon Vary

# Why direct imaging?

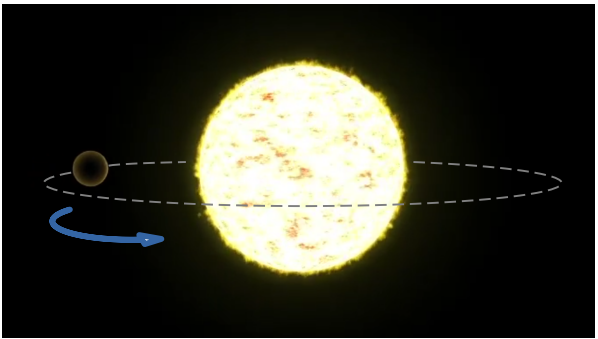
- Disadvantages of indirect methods

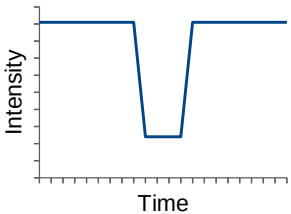




# Why direct imaging?

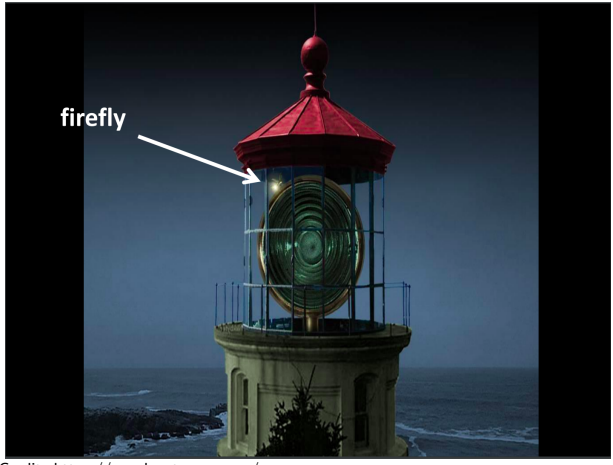
- Disadvantages of indirect methods






- Promising to characterize the atmospheres of exoplanets

# Direct Imaging




Credit: https://exoplanets.nasa.gov/

# Direct Imaging

Credit: https://exoplanets.nasa.gov/

- firefly $\rightarrow$ exoplanet
- lighthouse $\rightarrow$ star

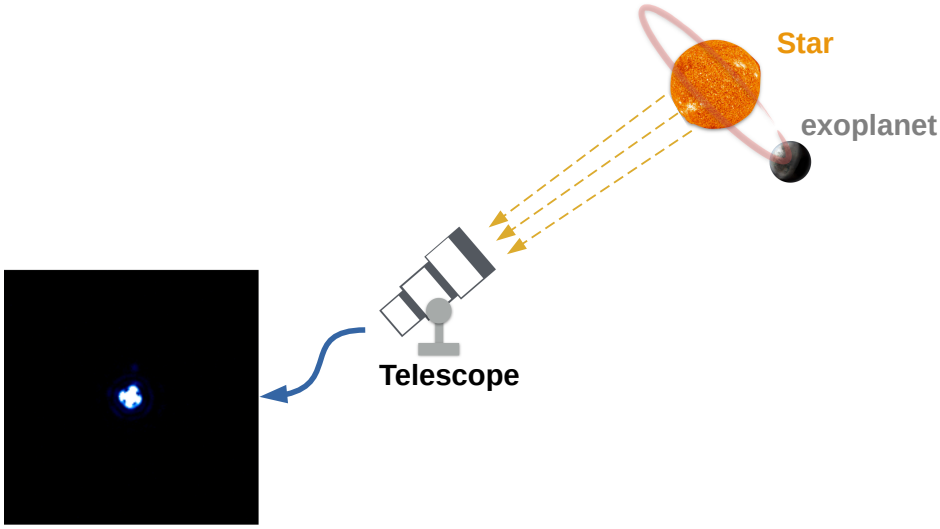
# Direct Imaging

Credit: https://exoplanets.nasa.gov/

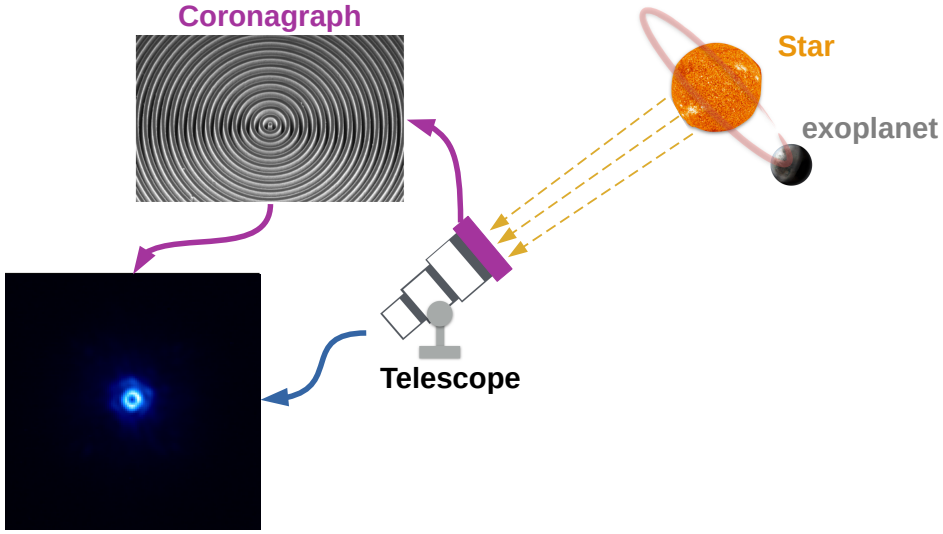
- firefly $\rightarrow$ exoplanet
- lighthouse $\rightarrow$ star

$$10^3 \leq \frac{\text{Intensity of star}}{\text{Intensity of exoplanet}} \leq 10^{10}$$

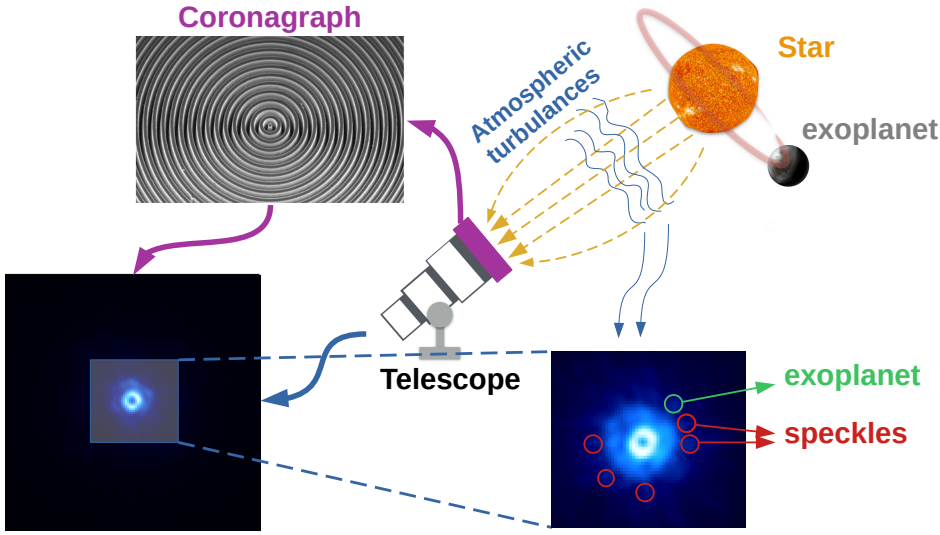
# Direct Imaging



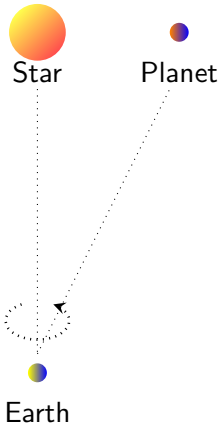
# Direct Imaging

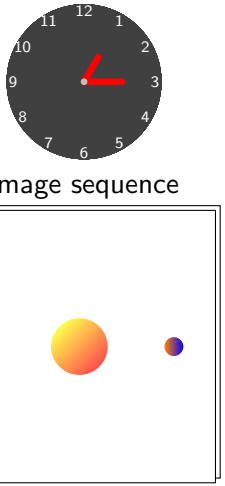


# Direct Imaging

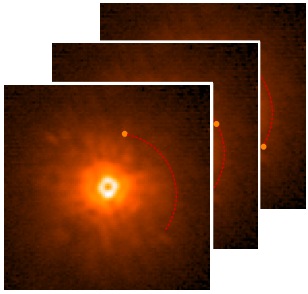


# Angular Differential Imaging



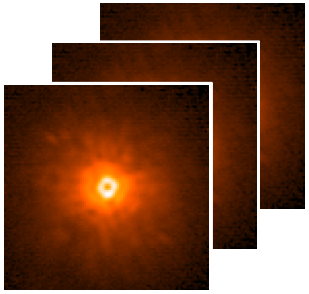



# Goal



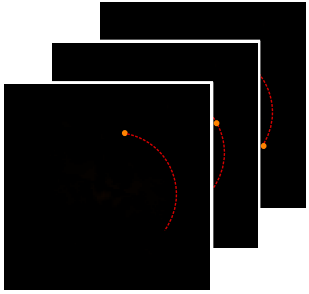
Original cube

=



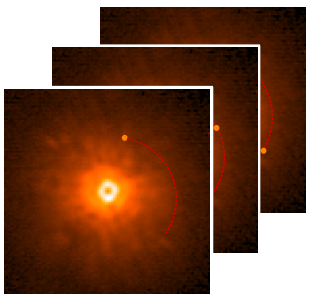
Background

+



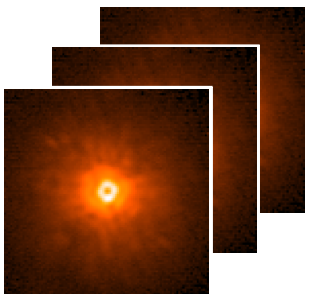
Foreground

# Goal



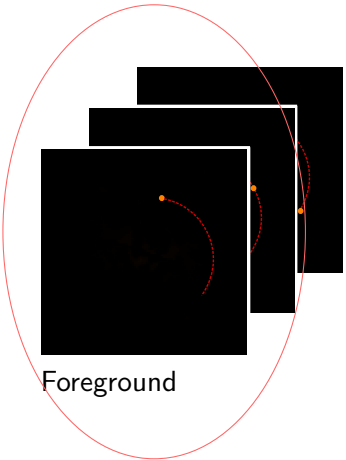
Original cube

=



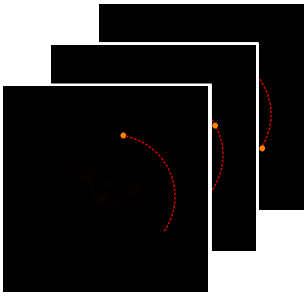
Background

+



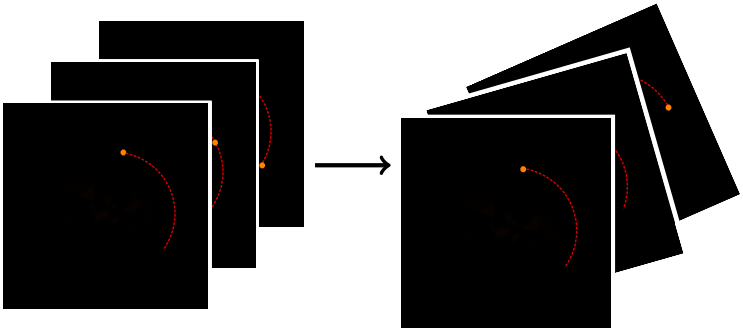
Foreground

# Goal



Foreground

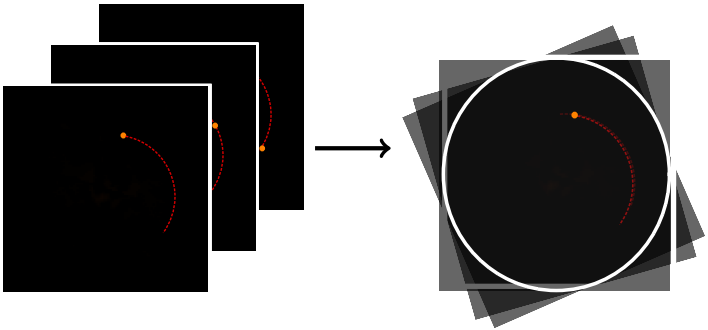
# Goal



Foreground

Derotate the frames

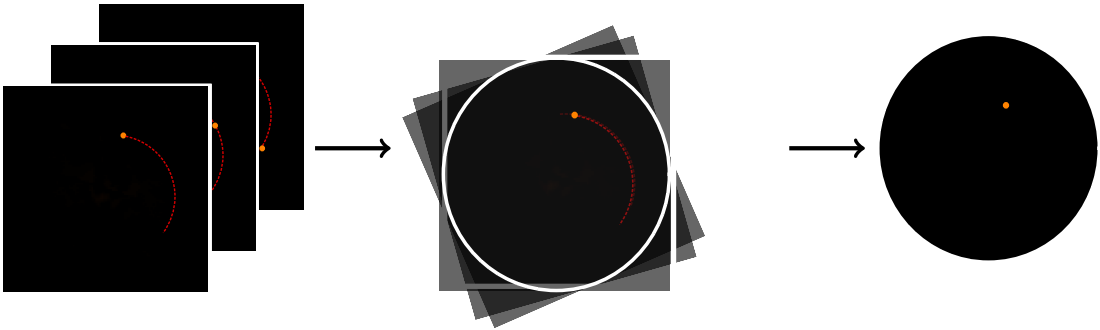
# Goal



Foreground

Derotate the frames

# Goal

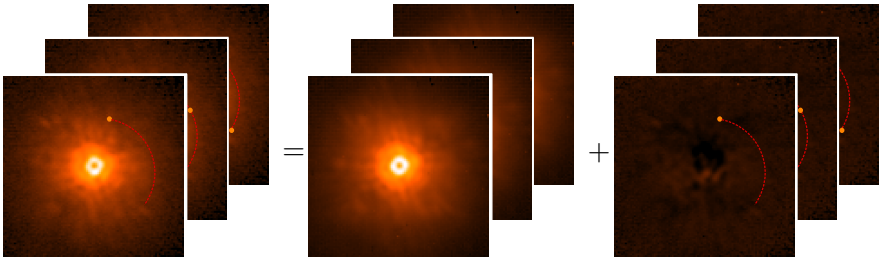


Foreground

Derotate the frames

Mean/Median frame

# State of art: PCA[1,2]



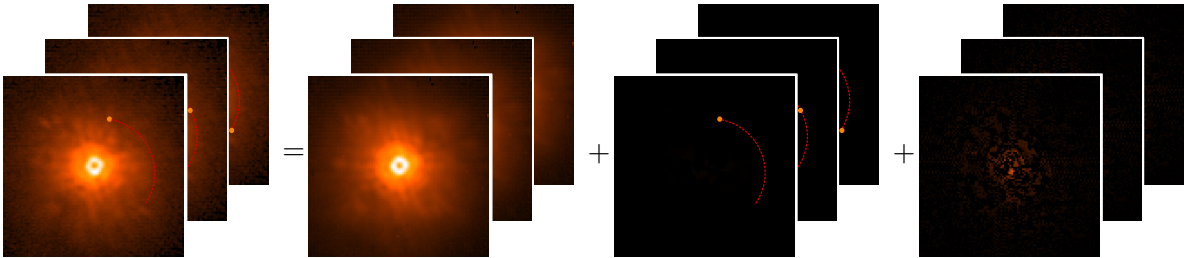
Original cube
$M$

Low rank
$L$

Foreground
$G$

[1] Amara and Quanz, 2012
[2] Soummer, et al., 2012

# State of art: LLSG[3]



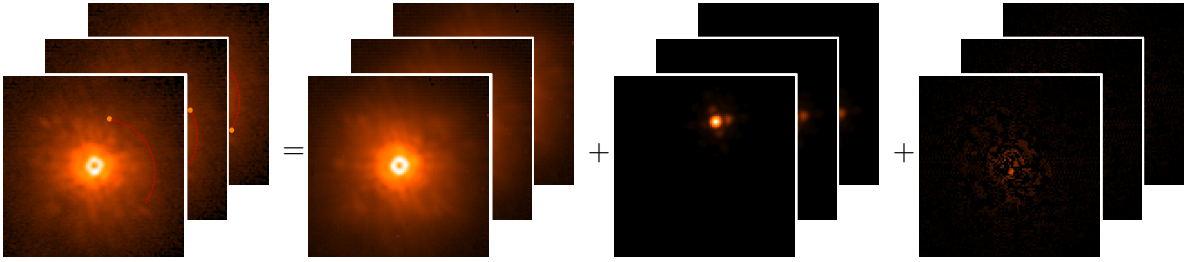
Original cube $M$ = Low rank $L$ + Sparse $S$ + Gaussian $G$

$$\mathrm{rank}(L) \leq k, \quad \mathrm{card}(S) \leq s$$

[3] Gomez Gonzalez, et al., 2016

How can we obtain $S$ ?

# Minimization with Trajectory



Original cube
$M$

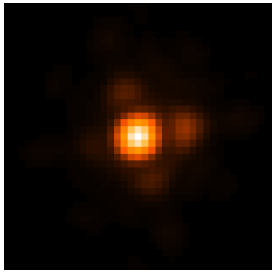
Low rank
$L$

Planet signature
$aP_g$

Gaussian
$G$

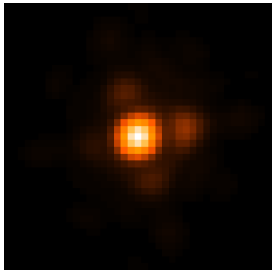
$$\operatorname{rank}(L) \leq k, \quad P_g \in \Lambda$$

# Planet signature

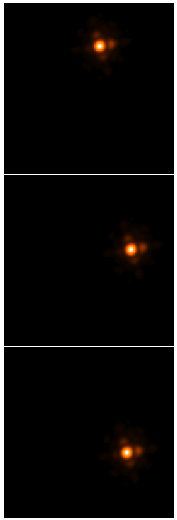


PSF

# Planet signature

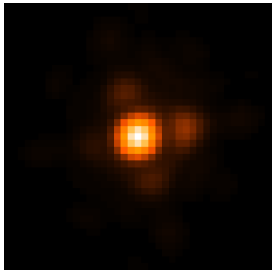


PSF

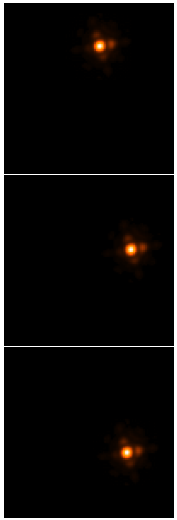


Copy PSF
along the trajectory $g$

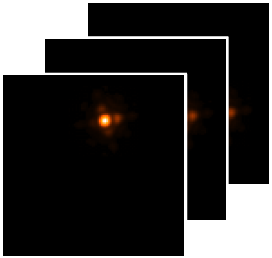
# Planet signature



PSF



Copy PSF
along the trajectory $g$



Planet signature
$P_g$

## Minimization with trajectory

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\|_F \quad \text{s.t.} \quad \operatorname{rank}(L) \leq k$$

- Solve (1) by SVD
- Solve (2) by

$$a_i = \frac{\langle P_g, M - L_i \rangle}{\|P_g\|_F^2}$$

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$$L_i = _{L\in\mathbb{R}^{t\times n}} \|M - L - a_{i-1}P_g\|_F^2 \qquad (1)$$

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- Solve (1) by SVD
- Solve (2) by

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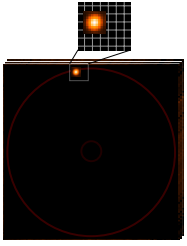
# Trajectories

Using $P_g$



Solve

**Minimization with trajectory**

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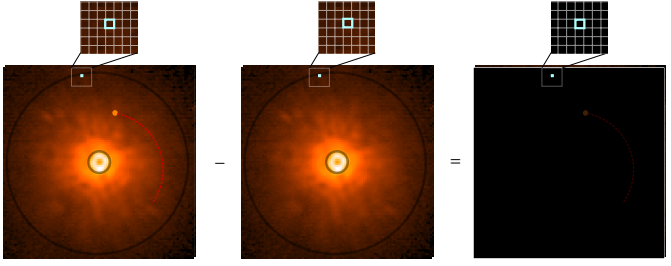


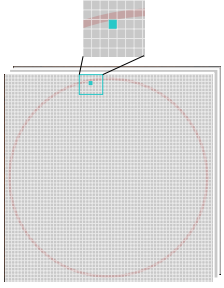

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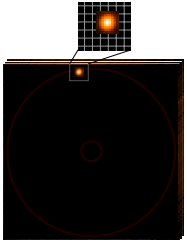
Using $P_g$



Solve

**Minimization with trajectory**

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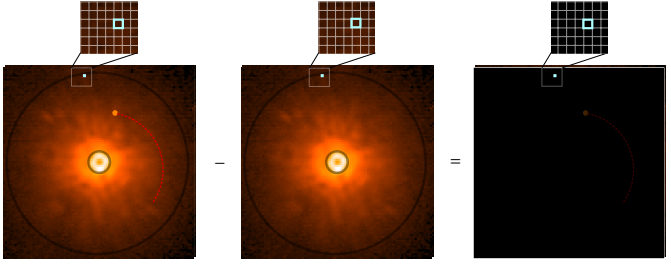
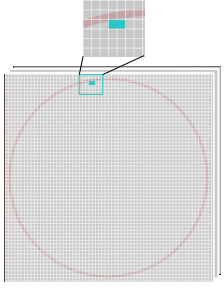
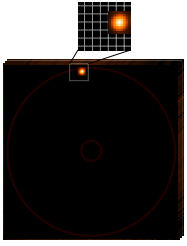


# Trajectories

Using $P_g$



Solve

**Minimization with trajectory**

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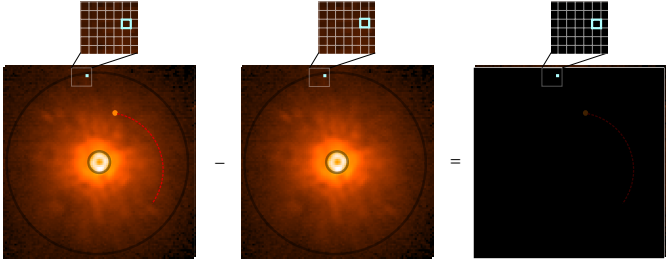
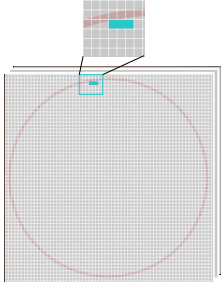
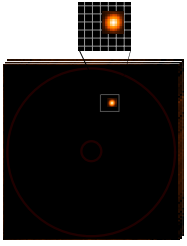


# Trajectories

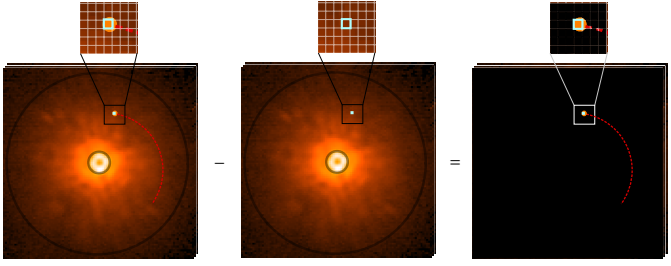
Using $P_g$

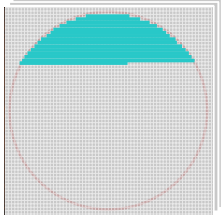


Solve

**Minimization with trajectory**

$$\min_{L \in \mathbb{R}^{t \times n}, a \in \mathbb{R}} \|M - L - aP_g\|_F \quad \text{s.t.} \quad \operatorname{rank}(L) \leq k$$

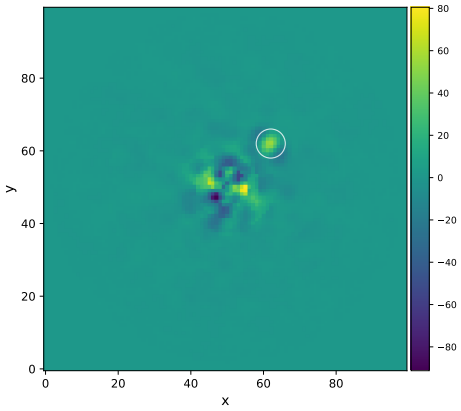




# Detection Maps

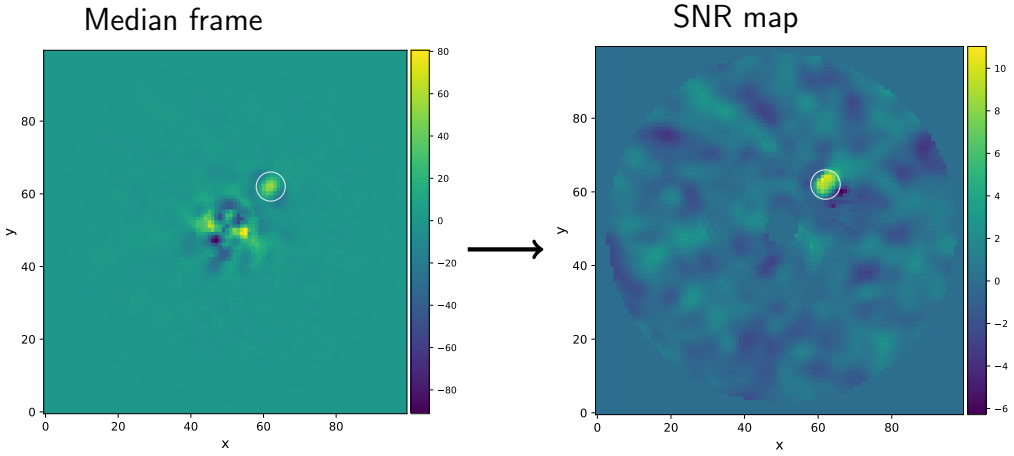
**Real planet:**

Median frame



# Detection Maps

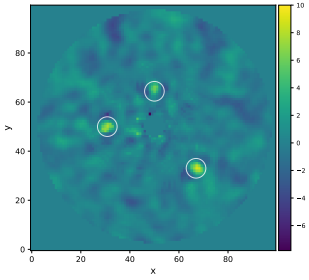
**Real planet:**



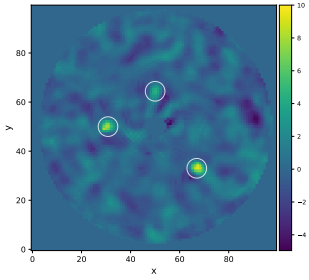
# Detection Maps

**Fake planets:**

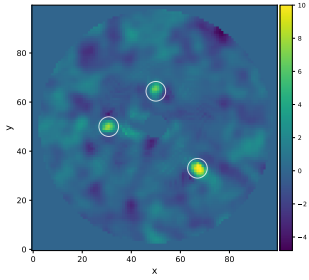
PCA



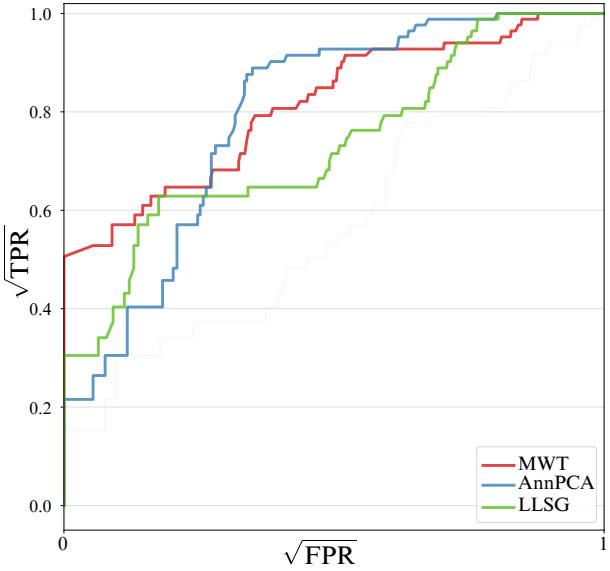

LLSG




MWT



# ROC Curve Comparison



Thank you for your attention!
Any questions?